附 件

一、理论分析部分模型求解

（1）模型设定

行业需求函数：$P_S = \frac{\theta_s}{Y_S} PY$

企业需求函数：$\mathrm{P_{si}} = P_s\left(\frac{Y_s}{Y_{si}}\right)^{1/\sigma}$

企业最优化问题：

$$\max_{P_{si},K_{si},L_{si},\mathrm{B_{si}}} P_{si}Y_{si} - \frac{1}{1+\tau_{si}^K} rK_{si} - wL_{si} - B_{si}$$

满足约束条件：

$$P_{si} = P_s\left(\frac{Y_s}{Y_{si}}\right)^{\frac{1}{\sigma}} \tag{1}$$

$$Y_{si} = A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s},\ 0<\alpha_s<1 \tag{2}$$

$$1+\tau_{si}^K = (Z_{si}B_{si})^{\beta_s^K} \tag{3}$$

企业最优化问题的一阶条件：

$$MRPK_{si} \equiv \alpha_s \frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{K_{si}} = \frac{1}{1+\tau_{si}^K} r \tag{4}$$

$$MRPL_{si} \equiv \left(1-\alpha_s\right)\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{L_{si}} = w \tag{5}$$

$$\beta_s^K \frac{1}{\left(1+\tau_{si}^K\right)B_{si}} rK_{si} = 1 \tag{6}$$

（2）模型求解

联立企业最优化问题的一阶条件（1）-（3），可以分别得到要素投入的需求函数：

$$K_{si} = \left(\alpha_s \frac{\sigma-1}{\sigma}P_s\right)^{\sigma}\left(\frac{1+\tau_{si}^K}{r}\right)^{\sigma}\left(\frac{\left(1-\alpha_s\right)}{\alpha_s}\frac{r}{\left(1+\tau_{si}^K\right)w}\right)^{(1-\alpha)(\sigma-1)}\left(A_{si}\right)^{\sigma-1}Y_s \tag{7}$$

$$L_{si} = \left(\alpha_s P_s \frac{\sigma-1}{\sigma}\frac{1+\tau_{si}^K}{r}\right)^{\sigma}\left(\frac{\left(1-\alpha_s\right)}{\alpha_s}\frac{r}{\left(1+\tau_{si}^K\right)w}\right)^{\sigma-\alpha(\sigma-1)}\left(A_{si}\right)^{\sigma-1}Y_s$$

$$B_{si} = \left(\beta_s^K\right)^{\frac{1}{1-\alpha\beta_s^k(\sigma-1)}}\left(\alpha_s\right)^{\frac{1+\alpha(\sigma-1)}{1-\alpha\beta_s^k(\sigma-1)}}\left(\frac{\sigma-1}{\sigma}\right)^{\frac{\sigma}{1-\alpha\beta_s^k(\sigma-1)}}\left(\frac{1-\alpha_s}{w}\right)^{\frac{(1-\alpha)(\sigma-1)}{1-\alpha\beta_s^k(\sigma-1)}}\left(\frac{1}{r}\right)^{\frac{\alpha(\sigma-1)}{1-\alpha\beta_s^k(\sigma-1)}} \tag{8}$$

$$\times\left(P_s\right)^{\frac{\sigma}{1-\alpha\beta_s^k(\sigma-1)}}\left(Y_s\right)^{\frac{1}{1-\alpha\beta_s^k(\sigma-1)}}\left(Z_{si}\right)^{\frac{\alpha\beta_s^k(\sigma-1)}{1-\alpha\beta_s^k(\sigma-1)}}\left(A_{si}\right)^{\frac{\sigma-1}{1-\alpha\beta_s^k(\sigma-1)}} \tag{9}$$

在（9）式两边同时取对数，可以得到：

$$\log B_{si}=\frac{\sigma-1}{1-\alpha\beta_s^k\left(\sigma-1\right)}\log A_{si}+\frac{\alpha\beta_s^k\left(\sigma-1\right)}{1-\alpha\beta_s^k\left(\sigma-1\right)}\log Z_{si}+Constants \tag{10}$$

其中，$\gamma_s=\alpha\beta_s^k(\sigma-1)$。

（3）融资寻租弹性与资本错配

由（3）式可得：

$$Var\left[\log\left(1+\tau_{si}^K\right)\right]=\left(\beta_s^k\right)^2\left[\sigma_{Z_s}^2+Var\left(\log B_{si}\right)+2Cov\left(\log Z_{si},\log B_{si}\right)\right] \tag{11}$$

同时，由（10）式可得：

$$Var\left(\log B_{si}\right)=\left(\frac{\sigma-1}{1-\gamma_s}\right)^2\sigma_{A_s}^2+\left(\frac{\gamma_s}{1-\gamma_s}\right)^2\sigma_{Z_s}^2+2\left(\sigma-1\right)\gamma_s\left(\frac{1}{1-\gamma_s}\right)^2\sigma_{AZ_s} \tag{12}$$

$$Cov\left(\log B_{si},\log Z_{si}\right)=\frac{\sigma-1}{1-\gamma_s}\sigma_{AZ_s}+\frac{\gamma_s}{1-\gamma_s}\sigma_{Z_s}^2 \tag{13}$$

$$Cov\left(\log B_{si},\log A_{si}\right)=\frac{\sigma-1}{1-\gamma_s}\sigma_{A_s}^2+\frac{\gamma_s}{1-\gamma_s}\sigma_{AZ_s} \tag{14}$$

由（12）-（14）式可得：

$$\frac{\partial Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\partial\beta_s^k}=\frac{2Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\beta_s^k\left(1-\gamma_s\right)}\ldots 0 \tag{15}$$

关于融资寻租弹性与资本错配关系（15）式的证明如下：

首先由（11）式可知：

$$\begin{aligned}\frac{\partial Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\partial\beta_s^k}&=2\beta_s^k\left[\sigma_{Z_s}^2+Var\left(\log B_{si}\right)+2Cov\left(\log Z_{si},\log B_{si}\right)\right]\\&\quad+\left(\beta_s^k\right)^2\frac{\partial\gamma_s}{\partial\beta_s^k}\left[\frac{\partial Var\left(\log B_{si}\right)}{\partial\gamma_s}+2\frac{\partial Cov\left(\log Z_{si},\log B_{si}\right)}{\partial\gamma_s}\right]\end{aligned}$$

其中：

$$\begin{aligned}\frac{\partial Var\left(\log B_{si}\right)}{\partial\gamma_s}&=2\frac{1}{1-\gamma_s}\left(\frac{1}{1-\gamma_s}\right)^2\left[\left(\sigma-1\right)^2\sigma_{A_s}^2+\left(\gamma_s\right)^2\sigma_{Z_s}^2+2\left(\sigma-1\right)\gamma_s\sigma_{AZ_s}\right]\\&\quad+2\frac{1}{1-\gamma_s}\left[\frac{\gamma_s}{1-\gamma_s}\sigma_{Z_s}^2+\frac{\sigma-1}{1-\gamma_s}\sigma_{AZ_s}\right]\\&=\frac{2}{1-\gamma_s}\left[Var\left(\log B_{si}\right)+Cov\left(\log B_{si},\log Z_{si}\right)\right]\end{aligned}$$

$$\frac{\partial Cov\left(\log Z_{si},\log B_{si}\right)}{\partial \gamma_s}=\left(\frac{1}{1-\gamma_s}\right)^2\left[\left(\sigma-1\right)\sigma_{AZ_s}+\gamma_s\sigma_{Z_s}^2\right]+\left(\frac{1}{1-\gamma_s}\right)\sigma_{Z_s}^2$$
$$=\frac{1}{1-\gamma_s}\left[\frac{1}{1-\gamma_s}\left[\left(\sigma-1\right)\sigma_{AZ_s}+\gamma_s\sigma_{Z_s}^2\right]+\sigma_{Z_s}^2\right]$$
$$=\frac{1}{1-\gamma_s}\left[Cov\left(\log B_{si},\log Z_{si}\right)+\sigma_{Z_s}^2\right]$$

$$\frac{\partial Var\left(\log B_{si}\right)}{\partial \gamma_s}+2\frac{\partial Cov\left(\log Z_{si},\log B_{si}\right)}{\partial \gamma_s}=\frac{2}{1-\gamma_s}\left[Var\left(\log B_{si}\right)+Cov\left(\log B_{si},\log Z_{si}\right)\right]$$
$$+\frac{2}{1-\gamma_s}\left[Cov\left(\log B_{si},\log Z_{si}\right)+\sigma_{Z_s}^2\right]$$
$$=\frac{2}{1-\gamma_s}\left[Var\left(\log B_{si}\right)+2Cov\left(\log B_{si},\log Z_{si}\right)+\sigma_{Z_s}^2\right]$$
$$=\frac{2}{1-\gamma_s}\frac{Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\left(\beta_s^k\right)^2}$$

$$\frac{\partial \gamma_s}{\partial \beta_s^k}=\alpha\left(\sigma-1\right)$$

将$\frac{\partial \mathrm{Var}(\log \mathrm{B_{si}})}{\partial \gamma_s}$、$\frac{\partial \mathrm{Cov}(\log \mathrm{Z_{si}},\log \mathrm{B_{si}})}{\partial \gamma_s}$和$\frac{\partial \gamma_s}{\partial \beta_s}$的表达式代入$\frac{\partial \mathrm{Var}[\log(1+\tau_{si}^K)]}{\partial \gamma_s}$即可得到：

$$\frac{\partial Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\partial \beta_s^k}=\frac{2Var\left[\log\left(1+\tau_{si}^K\right)\right]}{\beta_s^k\left(1-\gamma_s\right)}\ldots 0$$

二、附表

表 1　　两位码行业资本收入份额

行业代码	资本收入份额
13	0.80
14	0.80
15	0.80
17	0.63
18	0.62
19	0.61
20	0.58
21	0.62
22	0.72
23	0.57
24	0.63
25	0.89

26	0.85
27	0.67
28	0.85
29	0.65
30	0.69
31	0.68
32	0.68
33	0.61
34	0.67
35	0.67
36	0.69
37	0.69
38	0.67
39	0.71
40	0.67
41	0.63
43	0.63